LaTeX Examples

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1 Two Theorems About Sums

Theorem. *The sum of two even integers is an even integer.*

Proof. Let m and n be any two even integers, which means $m = 2j$ and $n = 2k$ for some integers j and k. Now,

$$\begin{aligned} m + n &= 2j + 2k \\ &= 2(j + k). \end{aligned}$$

Since the integers are closed under addition, $j + k$ is an integer, and so we have written $m + n$ as twice some integer. Therefore, $m + n$ is even, as desired. □

Theorem. *The sum of any two rational numbers is rational.*

Proof. Let $r, s \in \mathbb{Q}$, which means $r = \frac{a}{b}$ and $s = \frac{c}{d}$, where $a, b, c, d \in \mathbb{Z}$ and $b, d \neq 0$.

Now,

$$\begin{aligned} r + s &= \frac{a}{b} + \frac{c}{d} \\ &= \frac{ad}{bd} + \frac{bc}{bd} \\ &= \frac{ad + bc}{bd}. \end{aligned}$$

By properties of integer addition and multiplication, we know $ad+bd, bd \in \mathbb{Z}$. We know also $bd \neq 0$, since $b, d \neq 0$. Therefore, $r + s$ is a rational number. □

2 A Ramble About e

It is an indisputable fact that e is a better number than π. Anyone who thinks otherwise is wrong.

Consider, for example,

$$\begin{aligned}\frac{d}{dx}(a^x) &= \lim_{h\to 0}\frac{a^{x+h}-a^x}{h}\\ &= a^x \lim_{h\to 0}\frac{a^h-1}{h}.\end{aligned}$$

This limit exists for all $a > 0$, but is there a choice that makes the limit equal unity? If this were possible, we would have

$$\lim_{h\to 0}\frac{a^h-1}{h} = 1,$$

which rearranges to

$$a = \lim_{h\to 0}(1+h)^{\frac{1}{h}}.$$

This limit exists and is roughly equal to 2.71828. The precise value of the limit is transcendental and is referred to by e (probably for "exponential").

According to our analysis, e^x is the only function (up to a constant factor) that is its own derivative. Based on that fact, the theory of Taylor series gives us another representation of e^x, namely

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}.$$

Take the formal derivative of the series and you'll see what I mean.

Finally, it is the only exponential function whose tangent line at the origin has unit slope.

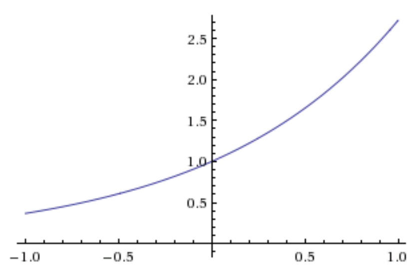

Figure 1: The graph of e^x is truly exquisite.