Karissa Vandenberg
Bailey Waite
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11.11

13. $f(x) = \sqrt{x}$, $a = 4$, $n = 2$, $4 \le x \le 4.2$

a) Approx f by a Taylor Polynomial with degree n at the #a.

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{4}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{32}$
3	$\frac{3}{8}x^{-5/2}$	$\frac{3}{256}$

$$\sqrt{x} \approx 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 = T_2(x)$$

b) Use Taylor's Inequality to estimate the accuracy of the approx. $f(x) \approx T_n(x)$ when x lies in the given interval.

$$|R_2(x)| \le \frac{f^{(3)}(x)}{3!}|x-4|^3$$
$$= \frac{3/8\,(x)^{-5/2}}{3!}|x-4|^3$$

Bound $|x-4|^3$ on $4 \le x \le 4.2$

Biggest at $x = 4.2$ $\quad |4.2-4|^3 = 0.008$

Madi Montgomery
LeighAnn
Dow

11.11 Practice Problems

13. $f(x) = \sqrt{x}$

$a = 4$

$n = 2$

$4 \le x \le 4.2$

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{4}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{32}$

$$\sqrt{x} \approx \underbrace{2}_{n=0} + \underbrace{\tfrac{1}{4}|x-4|}_{n=1} - \underbrace{\tfrac{1}{64}|x-4|^2}_{n=2} = T_2(x)$$

$$|R_2(x)| \le \frac{f^{(3)}(x)}{3!}|x-4|^3$$

$$\le \frac{\frac{3}{8}x^{-5/2}}{6}|x-4|^3$$

$|x-4|^3$ on $4 \le x \le 4.2$

$|4.2-4|^3 = .008$

$|x-4|^3 \le .008$ on $4 \le x \le 4.2$

$\left|\frac{3}{8x^{5/2}}\right|$ on $4 \le x \le 4.2$

$\left|\frac{3}{8x^{5/2}}\right| \le \frac{3}{8(4)^{5/2}} < .0117$

$$|R_2(x)| \le \frac{.0117}{3!} \cdot .008$$

$$\boxed{\le .0000156}$$

21. $f(x) = x \sin x$

$a = 0$

$n = 4$

$-1 \le x \le 1$

Andrew Maaske
Alli Kolal
Seth Lindgren

13) $f(x) = \sqrt{x}$ $\quad a=4 \quad n=2 \quad 4 \le x \le 4.2$

a)

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{2}(4)^{-1/2} = \frac{1}{4}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{4}(4)^{-3/2} = -\frac{1}{32}$
3	$\frac{3}{8}x^{-5/2}$	

$$\sqrt{x} \approx \boxed{2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2} = T_2(x)$$

b) $|R_2(x)| \le \dfrac{\left|\frac{3}{8}x^{-5/2}\right|}{6}|x-4|^3$

Bound $|x-4|^3$ on $4 \le x \le 4.2$?

Biggest at $x = 4.2$

$|4.2-4|^3 = 0.008$

$|x-4|^3 \le 0.008$ on $4 \le x \le 4.2$

Bound $\left|\frac{3}{8}x^{-5/2}\right|$ on $4 \le x \le 4.2$?

$\left|\dfrac{3}{8x^{5/2}}\right| \le \dfrac{3}{8 \cdot 4^{5/2}} < 0.01171875$

$$|R_2(x)| \le \frac{0.01171875}{6} \cdot 0.008 < \boxed{0.000015625}$$

Riley S.
Jaret P.

11.11 4-20-15

(13)

n	$f^n(x)$	$f^n(4)$
0	$\sqrt{x}$	2
1	$\frac{1}{2}x^{-\frac{1}{2}}$	$\frac{1}{8}$
2	$-\frac{1}{4}x^{-\frac{3}{2}}$	$-\frac{1}{32}$
3	$\frac{3}{8}x^{-5/2}$	

$$R_2(x) = \frac{\left|\frac{3}{8}x^{-5/2}\right|}{6}|x-4|^3$$

(B)

Bound $|x-4|^3$ on $4 \le x \le 4.2$

$|.2|^3 = .008$

Bound $\frac{3}{8}x^{-5/2}$ on $4 \le x \le 4.2$

$$R_2(x) \le \frac{.0117}{6}|.008|$$

$$\boxed{= 1.56 \times 10^{-5}}$$

(A) $\sqrt{x} \approx 2 + \frac{1}{8}(x-4) - \frac{1}{64}(x-4)^2$

(21)

n	$f^n(x)$	$f^n(0)$
0	$x \sin x$	
1		
2		
3		
4		
5		

$-\frac{1}{2}-\frac{2}{2}$

Sydni Matt

13. $f(x)=\sqrt{x}$ $a=4$ $n=2$ $4\le x\le 4.2$

n	$f^{(n)}(x)$	$f(4)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{2}(4)^{-1/2}=\frac{1}{8}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{4}(4)^{-3/2}=-\frac{1}{32}$
3	$\frac{3}{8}x^{-5/2}$	$\frac{3}{8}(4)^{-5/2}$

$R_2(x)=\frac{\left|\frac{3}{8}(x)^{-5/2}\right|}{3!}\,|x-4|^3$

B. $\left|\frac{3}{8}(x)^{-5/2}\right|$ $4\le x\le 4.2$

$x=4$

$|x-4|^3$

$x=4.2$

$\frac{\left|\frac{3}{8}(4)^{-5/2}\right|}{3!_{6}}\cdot(4.2-4)^3 < 1.56\times10^{-5}$

A $\sqrt{x}\approx \underset{n=0}{2}+\underset{n=1}{\frac{1}{8}(x-4)}-\underset{n=2}{\frac{1}{64}(x-4)^2}$

21. $f(x)=x\sin x$ $a=0$ $n=4$ $-1\le x\le 1$

n	$f(x)$
0	$x\sin x$
1	$\sin x\cdot x\cos x$
2	
3	
4	
5	

Trip, Travis, Taylor

13) $f(x) = \sqrt{x}$ $a = 4$ $n = 2$ $4 \le x \le 4.2$

a) $\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$\sqrt{x}$	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{4}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{4}\left(\frac{1}{8}\right) = -\frac{1}{32}$
3	$+\frac{3}{8}x^{-5/2}$	

$\sqrt{x} \approx 2 + \frac{\frac{1}{4}}{1!}(x-4)^1 + \frac{-\frac{1}{32}}{2!}(x-4)^2$

b) $|R_n(x)| \le \frac{|f^{n+1}(x)|}{(n+1)!}|x-a|^{n+1}$

$|R_2(x)| \le \frac{|\frac{3}{8}x^{-5/2}|}{3!}|x-4|^3$

Bound $|x-4|^3$ on $4 \le x \le 4.2$

max at 4.2

$.2^3$ $.008$

Bound $\frac{3}{8x^{5/2}}$ $4 \le x \le 4.2$

max a 4

$|R_2(x)| \le .000016$

21) $f(x) = x\sin(x)$ $a = 0$ $n = 4$ $-1 \le x \le 1$

$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$

n	$f^n(x)$	$f^n(0)$
0	$x\sin(x)$	0
1	$x\cos(x) + \sin(x)$	$0 + 0 = 0$
2	$-\sin(x)x + \cos(x) + \cos(x)$	
3		
4		
5		

(13) $f(x) = \sqrt{x}$ Patrick, An, Nicole $a=4$ $n=2$ $4 \le x \le 4.2$

a) $F(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} |x-a|^n$

n	$f^n(x)$	$f^n(a)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	$1/4$
2	$-\frac{1}{4}x^{-3/2}$	$-1/32$
3.	$\frac{3}{8}x^{-5/2}$	

$$\boxed{2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2}$$

b.) $|R_2(x)| \le \frac{|f^{(n+1)}(x)|}{3!} (x-4)^3$

$\frac{\frac{3}{8}x^{-5/2}}{6}$ $\frac{1}{125}$ $\left(\frac{1}{5}\right)^3 \cdot \frac{1}{125} = \max(x-4)^3$

$\max\left(\frac{3}{8}x^{-5/2}\right)$ $\frac{3}{8}\left(\frac{1}{4^{5/2}}\right.$ $\frac{3}{8} \cdot \frac{1}{32} = \frac{3}{256}$

$\frac{\frac{3}{256}}{6} \cdot \frac{1}{125}$ $R_2(x) \le \frac{3}{256} \cdot \frac{1}{125} \cdot \frac{1}{6}$

$$\boxed{R_2(x) \le 1.56 \times 10^{-5}}$$

(21) $f(x) = x\sin x$ $a=0$ $n=4$ $-1 \le x \le 1$

a) $\frac{2}{2!}(x-0)^2 + \frac{4}{4!}(x-0)^4$

$$\boxed{x^2 + \frac{1}{6}x^4}$$

n	$f^n(x)$	$f^n(0)$
0	$x\sin x$	0
1	$\sin x + x\cos x$	0
2	$-x\sin x + 2\cos x$	2
3	$-x\cos x - 3\sin x$	0
4	$x\sin x - 4\cos x$	4