Trigonometric Identities

Pythagorean Identities

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ 1 + \tan^2\theta &= \sec^2\theta\\ 1 + \cot^2\theta &= \csc^2\theta\end{aligned}$$

Half-Angle Identities

$$\begin{aligned}\sin^2\theta &= \frac{1}{2}(1 - \cos(2\theta))\\ \cos^2\theta &= \frac{1}{2}(1 + \cos(2\theta))\end{aligned}$$

Error Bounds for Approximate Integration

Let E_M and E_T denote the error in estimating an integral with the Midpoint and Trapezoid Rules, respectively. The use of these rules to evaluate $\int_a^b f(x)\,dx$ on n subintervals yields

- $|E_M| \leq \dfrac{K(b-a)^3}{24n^2}$ and
- $|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$,

where K is a constant satisfying $|f''(x)| \leq K$.

Hydrostatic Force

The hydrostatic force of water against a surface with an area of A square meters at a depth of d meters is approximately $9800dA$ Newtons.

Taylor Series

The Taylor series for the function f centered at a is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Common Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$

Vector Products

Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$. The dot product and cross product, respectively, are given by:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$