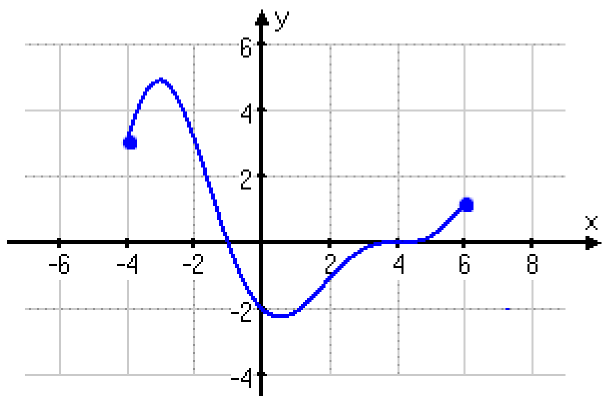
Name: ______________________________

Please write *only* your name on the test sheet.
Place all work and answers on the blank sheets provided.
All items are equally weight unless otherwise noted.

1. Refer to the graph below for the following questions. For questions asking you to identify specific coordinates, make your best estimate based on the graph.



   (a) Is this the graph of a function? Why or why not?

   (b) What are the x-intercept(s) of this relation? What are the y-intercept(s)?

   (c) On what x-interval(s) is the relation increasing?

   (d) What are the *coordinates* of the global maximum?

   (e) What is the average rate of change between $x = -4$ and $x = 6$?

   Solution:

   (a) There is at most one output for every input, as evidenced by the vertical line test. The relation presented in the graph is therefore a function.

   (b) The graph of the function crosses the x-axis at $(-1, 0)$ and $(4, 0)$. It crosses the y-axis at $(0, -2)$.

   (c) The function increases over the x-intervals $(-4, -3)$ as well as $(1, 6)$ (approximately).

   (d) The global maxium is the point with the largest y value, which occurs at $(-3, 5)$.

   (e) The average rate of change is given by

$$\frac{f(6) - f(-4)}{6 - (-4)} = \frac{1 - 3}{6 + 4} = -0.2.$$

2. The demand for wheat is modeled by $D(p) = -0.76p + 3.12$, where $D(p)$ is the number of bushels sold (in millions) when the price per bushel is p dollars.

   (a) What price should be charged in order to sell a total of two million bushels?

(b) What is the y-intercept of the demand function and what does it represent *specifically* in this context?

(c) What is the slope of the demand function and what does it represent *specifically* in this context?

(d) The supply of wheat is modeled by $S(p) = 1.43p + 1.56$, where p is the price per bushel and $S(p)$ is the number of bushels produced (in millions). Determine the equilibrium price and quantity according to $D(p)$ and $S(p)$.

Solution:

(a) To determine this price, we must solve $2 = -0.76p + 3.12$ for p, which gives $p \approx 1.47$. We should therefore charge \$1.47.

(b) The y-intercept is $(0, 3.12)$, which represents the fact that we will sell 3.12 million bushels of wheat if we charge nothing for them.

(c) The slope is -0.76 million bushels per dollar, which represents the fact that each increase in price of \$1 reduces the number of bushels sold by 760,000 (0.76 million).

(d) To determine the equilibrium price, we need to solve $1.43p + 1.56 = -0.76p + 3.12$, which gives $p \approx 0.71$. For this value of p, the supply and demand function both yield approximately 2.58. This means the equilibrium price is \$0.71 and the equilibrium quantity is 2.58 million bushels.

3. The table below gives sparse data about the height $h(x)$ (in inches) of an infant measured x months after its birth.

Month x	Height $h(x)$ (in.)
2	25.8
4	26.6
6	27.4
8	28.2

(a) Why is a linear model appropriate for this data?

2x (b) Write a linear model for $h(x)$.

Solution:

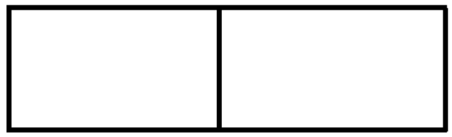
(a) The average rate of change between the first two points of data is given by

$$\frac{26.6 - 25.8}{4 - 2} = 0.4 \text{ inches per month.}$$

Proceeding similiary for the other pairs of consecutive data, we see that the average rate of change is 0.4 inches per month throughout. A constant rate of change is a defining property of linear functions.

(b) We computed the slope to be 0.4 in part (b), so the function looks like $f(x) = b + 0.4x$. To determine the initial value, we can substitue any point of data and solve for b. Choosing the point $(2, 25.8)$ gives $25.8 = b + 0.4 \cdot 2$, which shows $b = 25$. The linear function is therefore $f(x) = 25 + 0.4x$.

2x 4. A rectangular field is fenced and further divided into two pens of equal area as shown below. Construct a *single-variable* model for the area of the pen assuming that there is 100 feet of fencing available to use.



Solution: Let w denote the horizontal width of the entire fenced rectangle and h denote the vertical height. The area of the two pens can be expressed as a function of two variables $A(w, h) = wh$. We can make use of the constraint that the perimeter is 100 feet with the equation $100 = 2w+3h$. Solving this for w (we could choose either variable) gives $w = 50 - 1.5h$. This fact allows us to reduce the area function down to a single variable $A(h) = (50 - 1.5h)h$, or equivalently $A(h) = 50h - 1.5h^2$.