

I will let you know if the questions this year differ significantly from the following.

- Test 1: Problems 1 – 4
- Test 2: Problems 5 – 8
- Test 3: Problems 9 – 12
- Test 4: Problems 13 – 15

1. Let  $p$  denote the price of a certain cell phone. The supplier is willing to produce phones according to  $S(p) = 5p - 200$  (in millions). Consumer demand is given by  $D(p) = 900 - 3p$  (in millions).
  - (a) What price must be charged if the company wishes to produce 100,000,000 phones?
  - (b) How many phones will be purchased if the price is \$100?
  - (c) At what price does supply equal demand? How many phones will be sold at this price?
2. The following table shows the temperature (in degrees Fahrenheit)  $t$  hours after dawn on a certain day.

<b>Hour</b>	1	3	5
<b>Temp.</b>	50	53	56

- (a) How does the data suggest that a **linear** model is appropriate?
  - (b) Construct a linear function  $T(t)$  that models the temperature (in degrees Fahrenheit)  $t$  hours after dawn.
3. (a) Solve the following system of equations **by hand** using Gauss-Jordan reduction. (There will be infinitely-many solutions.)

$$\begin{aligned}2x - y - z &= 0 \\ -x - y + 2z &= 0 \\ -x + 2y - z &= 0\end{aligned}$$

- (b) Solve the following system of equations using matrix algebra and a **calculator**. (The solution will be unique.)

$$\begin{aligned}x + y + 6z &= -1 \\ \frac{1}{3}x - \frac{1}{3}y + \frac{2}{3}z &= 1 \\ \frac{1}{2}x + z &= 0\end{aligned}$$

- (c) Your company buys paper by the palette from three different suppliers: Red Co., Green Ltd., and Blue Inc. For one palette, the suppliers charge \$85, \$50, and \$65, respectively. Last month, you ordered a total of 100 palettes and spent \$5,990. You also ordered the same number from Red Co. as you did from Blue Inc. Construct, **but do not solve**, a system of equations that would determine the number of palettes ordered from each supplier.
4. Write a **single** matrix equation to compute the requested information. Be sure to clearly define the matrices you use.
- (a) The following tables summarize sales information for three different bookstores.

**January**

	Los Angeles	San Francisco	San Diego
hardback	300	200	150
paperback	400	250	200

**February**

	Los Angeles	San Francisco	San Diego
hardback	300	250	100
paperback	450	300	150

- What is the total number of sales of each type of book at each store from January to February? (Your answer should be a single matrix with six entries.)
- (b) Refer back to the February sales data. Suppose all the stores charge \$15 for a paperback book and \$25 for a hardback book. What is the total revenue at each store? (Your answer should be a single matrix with three entries.)
5. (a) Reduce the following payoff matrix by dominance.

$$\begin{bmatrix} 2 & -4 & 9 \\ 1 & 1 & 0 \\ -1 & -2 & -3 \\ 1 & 1 & -1 \end{bmatrix}$$

- (b) You run a TV station. At midnight, you can air either a reality show or a movie. A rival station will run either a nature documentary or a ballet at that time. If you run the reality show against the nature documentary, you will lose 2,000 viewers. If you run the reality show against the ballet, you experience no change in viewership. If you run the movie against the nature documentary, you will gain 2,000 viewers. If you run the movie against the ballet, you will lose 1,000 viewers. Supposing you play this game many times, what is your safest strategy according to the minimax criterion?
6. Two sectors of the U.S. economy are (1) crude petroleum and natural gas and (2) petroleum refining and related industries. According to government figures, in 1999 the crude sector used \$20,000 million of its own resources and sent \$55,000 million worth

of resources to the refined sector. The refined sector used \$20,000 million of its own resources and sent \$700 million worth of resources to the crude sector. That year, the crude sector produced a total of \$95,000 million worth of goods, and the refined sector produced a total of \$140,000 million worth of goods. Assuming unit costs stay the same, what total level of production should the sectors produce next year to meet a consumer demand of \$2,000 million worth of crude goods and \$150,000 million worth of refined goods?

7. (a) Solve the following linear program **by hand**. Include a sketch of the feasible region.

$$\begin{aligned} &\text{Maximize } x + 2y \\ &\text{subject to } x + 3y \leq 24 \\ &\quad 2x + y \leq 18 \\ &\quad x \geq 0 \\ &\quad y \geq 0 \end{aligned}$$

- (b) Give the objective function and constraints for the following scenario. Do **not** solve the linear program.

Your farm encompasses 100 acres and you are planning to grow tomatoes, lettuce, and carrots. Fertilizer costs per acre are: \$5 for tomatoes, \$4 for lettuce, and \$2 for carrots. Each acre of tomatoes will require 4 hours of labor per week, while lettuce and carrots each require 2 hours per week per acre. Profits per acre are: \$2,000 for tomatoes, \$1,500 for lettuce, and \$500 for carrots. You want to spend at least \$400 on fertilizer and you can supply up to 500 hours of labor per week. How many acres of each crop should you plant to maximize your total profit?

8. The following table shows the performance of a selection of stocks.

	Pharmaceutical (P)	Electronic (E)	Internet (I)	Total
Increased (V)	10	5	15	30
Unchanged (N)	30	0	10	40
Decreased (D)	10	5	15	30
Total	50	10	40	100

- (a) Describe, in words, the stocks that belong to the set  $D' \cap I$ . How many stocks belong to this set?
- (b) Using set notation, write an expression for the set of stocks that (1) did not increase or (2) increased and are electronic. How many stocks belong to this set?
9. (a) A bag contains 3 red marbles, 3 green ones, 2 yellow ones, and 2 orange ones. How many sets of five do **not** contain all the red marbles?
- (b) You throw a die five times and form a sequence by writing down the face for each throw. How many of the outcomes have exactly three 1s?

10. A red die and a green die are tossed (each having six sides).
- What is the probability that exactly one of the dice is even?
  - Are the following events independent?
    - The red die shows a 1.
    - The sum of the two dice is 5.
11. (a) A lie detector test misses only one lie out of 100, but 25% of people who are telling the truth fail the test anyway. In a particular screening, it is estimated that 1 out of 200 people have engaged in theft (and will therefore lie on the test). If the test indicates that a person is lying, what is the probability that they actually had engaged in theft?
- (b) The probability that a person in the United States declared bankruptcy in 2004 is 0.006. The probability that a person declared bankruptcy and had recently experienced a traumatic event is 0.005. What is the probability that a person who declared bankruptcy that year had recently experienced a traumatic event?
12. A credit card company classifies its cardholders as either “good” or “poor” standing. Each year, 75% of all cardholders with good standing remain good and 80% of all cardholders with bad standing remain bad. The remaining cardholders switch categories that year (i.e., they go from good to bad or bad to good).
- Suppose 25% of the customers currently have a poor credit rating. What percent of customers will have this rating in two years?
  - In the long term, what percent of customers will have a poor credit rating? (Give a formal justification using a system of equations.)
13. (a) A red die and a green die are tossed. Let  $X$  denote the difference of the two faces (the larger minus the smaller). Give the probability distribution of  $X$ .
- (b) A study shows that 25% of people connect to the Internet immediately upon waking. What is the probability that, in a randomly selected sample of ten people, exactly eight of them connect to the Internet immediately upon waking?
14. (a) Let  $X$  have the following probability distribution.
- |            |     |     |     |
|------------|-----|-----|-----|
| $x$        | 0   | 1   | 2   |
| $P(X = x)$ | 0.2 | 0.3 | 0.5 |
- Compute the mean of  $X$ .
- During a test, approximately 99% of all airbags functioned properly. Suppose we select 100 airbags at random and let  $X$  denote the number of them that functioned properly. Compute the variance of  $X$ .
15. SAT scores are normally distributed with mean 500 and standard deviation 100.
- What is the probability that a randomly chosen test-taker scored between 400 and 600?

- (b) What interval of **test scores** centered around the mean captures approximately 50% of all scores?