Test 1: Problems 1 – 4
Test 2: Problems 5 – 7
Test 3: Problems 8 – 10
Test 4: Problems 11 – 14

1 Lines and Planes

These two items will require you to use dot product (Section 12.3 Theorem 3) and cross product (Section 12.4 Theorem 5). I will provide the formula for computing both these products.

- Give vector and parametric equations of a line in space. (Section 12.5 Example 1 and 2)
- Give vector and linear equations of a plane. (Section 12.5 Example 4 and 5)

2 Cylinders and Quadric Surfaces

These two items will require you to identify a conic section from its equation (Section 10.5 Equation 1, 2, 4, 5, 7 and 8). Do not worry about minutia like directrix, focus, etc.

- Describe and sketch a cylinder from its equation. (Section 12.6 Example 1 and 2)
- Describe the traces of a quadric surface. (Section 12.6 Example 3, 4, 5, and 6)

3 Angle of Intersection and Arc Length

- Find the angle of intersection of two curves. (Section 13.2 Exercise 33, https://www.youtube.com/watch?v=ZbliSSiDVWc)
- Reparameterize a vector function with respect to arc length. (Section 13.3 Example 2)

4 Position, Velocity, and Acceleration

- Use derivatives and integrals, as appropriate, to translate between position, velocity, and acceleration. (Section 13.4 Example 1, 2, and 3)

5 Limits

- Prove a limit does not exist by exhibiting two paths of approach with different limiting behavior. (Example 14.2.2, 14.2.3)
- Prove a limit exists by appealing to continuity. (Example 14.2.5)
- Prove a limit exists using the Squeeze Theorem. (Example 14.2.4 - the note in the margin suggests how to use the Squeeze Theorem)

6 Partial Derivatives

- Use the Chain Rule to compute partial derivatives. (Example 14.5.3, 14.5.5)
- Compute directional derivatives. (Example 14.6.2, 14.6.4)
- Use the gradient to determine the direction and value of greatest change. (Example 14.6.6, 14.6.7)

7 Optimization

- Find extreme values of a function on a closed, bounded domain. (Example 14.8.3, 14.8.4)

8 Double Integrals

- Evaluate a double integral over a general region in Cartesian coordinates. (Example 15.3.1, 15.3.3, 15.3.4)
- Evaluate a double integral over a general region in polar coordinates. (Example 15.4.1, 15.4.4)

9 Triple Integrals

I will provide the conversion formulas to go from rectangular to cylindrical and rectangular to spherical.

- Set up, but do not evaluate, a triple integral in rectangular coordinates. (Example 15.7.2)
- Set up, but do not evaluate, a triple integral in cylindrical coordinates. (Example 15.8.1)
- Set up, but do not evaluate, a triple integral in spherical coordinates. (Example 15.8.4)

10 Change of Variables

- Set up, but do not evaluate, a double integral by applying an appropriate change of variables. (Example 15.9.3)

11 Line Integrals

This question will require you to parameterize a curve to evaluate the related integral. Relevant examples of constructing parameterizations are Examples 16.2.1 (polar), 16.2.2 (rectangular), and 16.2.6 (line segment). (Ignore the integrals themselves, since we did not discuss some of these. These examples are chosen only to demonstrate the parameterization of the curve.)

- Evaluate a line integral over a vector field using the definition of line integral. (Example 16.2.7)

12 Line Integral Theorems

- Evaluate a line integral over a conservative vector field using the fundamental theorem for line integrals. (Example 16.3.4)
- Evaluate a line integral over a vector field using Green's Theorem. (Example 16.4.1)

13 Surface Integrals

- Evaluate a surface integral over a vector field using the definition of surface integral. (Example 16.7.4, 16.7.5)

14 Surface Integral Theorems

The Stokes' Theorem question will require you to parameterize a surface to evaluate the related integral. Relevant examples of constructing parameterizations are Example 16.6.4 (spherical), 16.6.5 (cylindrical), and 16.6.6 (rectangular).

- Apply Stokes' Theorem to simplify, but not evaluate, a line integral. (Example 16.8.1)
- Apply the Divergence Theorem to simplify, but not evaluate, a surface integral. (Example 16.9.2)

Vector Products

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors. The dot product and cross product are given by:

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= u_1 v_1 + u_2 v_2 + u_3 v_3 \\ \mathbf{u} \times \mathbf{v} &= \langle u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1 \rangle \end{aligned}$$

Rectangular to Cylindrical

To convert from rectangular to cylindrical coordinates, use:

$$\begin{aligned} x &= r \cos\theta \\ y &= r \sin\theta \\ dV &= r\ dz\ dr\ d\theta \end{aligned}$$

Rectangular to Spherical

To convert from rectangular to spherical coordinates, use:

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \\ dV &= \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi \end{aligned}$$

Jacobian

Suppose the uv-plane is mapped to the xy-plane by the functions $x(u,v)$ and $y(u,v)$. The Jacobian is given by

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}.$$

Curl and Divergence

Let $\mathbf{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$ be a vector field. The curl and divergence of $\mathbf{F}$ are given by:

$$\begin{aligned} \operatorname{curl} \mathbf{F} &= \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle \\ \operatorname{div} \mathbf{F} &= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \end{aligned}$$