Test 1:	Problems 1 – 4
Test 2:	Problems 5 – 7
Test 3:	Problems 8 – 10
Test 4:	Problems 11 – 14

1. (a) Give the vector and parametric equations of the line through the points $(1, 1, 1)$ and $(2, 3, 4)$.

   (b) Give vector and linear equations for the plane containing the points $(1, 1, 1)$, $(1, 2, 3)$, and $(4, 5, 6)$

2. (a) Give a verbal description and rough sketch of the graph of $x^2 + 4z^2 = 4$ (in three-dimensional space).

   (b) Give a verbal description of the traces of $2x = y^2 + 2z^2$ in the planes $x = k$, $y = k$, and $z = k$ for arbitrary k. In each case, specify which values of k result in a trace at all.

3. (a) Reparameterize the curve $\mathbf{r}(t) = \langle 2t^3, -t^3, 3t^3 \rangle$ with respect to arc length measured from the origin in the direction of increasing t.

   (b) Determine the *point* in space where the curves $\mathbf{r}_1(s) = \langle s^2, s - 3, 2 - s \rangle$ and $\mathbf{r}_2(t) = \langle t^2 + 1, t - 2, t + 1, \rangle$ intersect and the angle of intersection.

4. (a) The position function of a particle is given by $\mathbf{r}(t) = \langle t^3 + 2t, 2t^2 - 1, t^4 - 3t^2 \rangle$. What is its acceleration function?

   (b) The velocity function of a particle is given by $\mathbf{v}(t) = \langle 2t, 3t, 2 \rangle$. Its position at time $t = 0$ is $\langle 1, 2, 3 \rangle$. What is its position function?

5. Compute each limit or demonstrate that it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{e^x}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + 2y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$

6. (a) Compute $\frac{\partial z}{\partial s}$ given that $z = x^3 + 3xy^3 + y^4$, $x = 2s - t$, and $y = st$.

   (b) The temperature at the point (x, y) on a metal plate is given by $T(x, y) = 3x^2y$ (in °C). What is the rate of change in temperature at the point $(1, 2)$ in the direction of the origin?

7. Find the absolute maximum and absolute minimum of $f(x, y) = xy^2$ on the domain $\{(x, y) \mid x^2 + y^2 \leq 4\}$. Clearly indicate *what* the extreme values are and *where* they occur. (Do **not** classify points as local maxima, local minima, or saddle points.)

8. Compute the volume of the specified solid using a double integral.
   (a) The solid under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$
   (b) The solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$

9. Set up, but **do not evaluate**, the following triple integrals. Setting up requires that you make the necessary conversion to the most appropriate coordinate system (rectangular, cylindrical, or spherical) and specify the bounds of integration on all three integrals.
   (a) $\iiint_E (x + 2y)\, dV$, where E is the solid bounded by the parabolic cylinder $y = x^2$ and the planes $x = z$, $x = y$, and $z = 0$
   (b) $\iiint_E x^2 + y^2\, dV$, where E is the solid bounded by the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the xy-plane
   (c) $\iiint_E e^z\, dV$, where E is the solid enclosed by the paraboloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$, and the xy-plane

10. Use the transformations $x = \frac{1}{4}(u + v)$ and $y = \frac{1}{4}(v - 3u)$ to simplify the integral

$$\iint_R 4x + 8y\, dA,$$

where R is paralellogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, and $(1, 5)$. **Do not evaluate** the resulting integral. Your answer should be a double integral in variables u and v with the bounds of integration specified on both integrals.

11. **Evaluate** $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the specified $\mathbf{F}$ and C using the definition of the line integral (i.e., do not apply any theorems).

    $\mathbf{F}(x, y) = \langle -y, x\rangle$ and C is the upper half of circle of radius 2 centered at the origin (Hint: Parameterize C using polar coordinates.)

12. **Evaluate** $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the specified $\mathbf{F}$ and C by applying an appropriate theorem.
   (a) $\mathbf{F}(x, y) = \langle 3 + 2xy, x^2 - 3y^2\rangle$, C is any path from the origin to the point $(1, 2)$
   (b) $\mathbf{F}(x, y) = \langle e^x + x^2y, e^y - xy^2\rangle$, C the positively-oriented square with vertices $(0, 0)$, $(1, 0)$ $(1, 1)$, and $(0, 1)$

13. **Evaluate** $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the specified $\mathbf{F}$ and S using the definition of the surface integral (i.e., do not apply any theorems).

    $\mathbf{F}(x, y, z) = \langle xy, yz, zx\rangle$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ above the square $0 \le x \le 1$, $0 \le y \le 1$

14. Apply an appropriate theorem to simplify, **but not evaluate**, each integral. Your answer should be a double or triple integral over a parameter domain with all bounds of integration specified.

(a) $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle yz, 2xz, e^{xy} \rangle$ and C is intersection of the cylinder $x^2 + y^2 = 16$ with the plane $z = 5$

(b) $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle x^2z^3, 2xyz^3, xz^4 \rangle$ and S is surface of the box with vertices $(\pm 1, \pm 2, \pm 3)$

Vector Products

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors. The dot product and cross product are given by:

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$$
$$\mathbf{u} \times \mathbf{v} = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

Rectangular to Cylindrical

To convert from rectangular to cylindrical coordinates, use:

$$x = r \cos \theta$$
$$y = r \sin \theta$$
$$dV = r \; dz \; dr \; d\theta$$

Rectangular to Spherical

To convert from rectangular to spherical coordinates, use:

$$x = \rho \sin \phi \cos \theta$$
$$y = \rho \sin \phi \sin \theta$$
$$z = \rho \cos \phi$$
$$dV = \rho^2 \sin \phi \; d\rho \; d\theta \; d\phi$$

Jacobian

Suppose the uv-plane is mapped to the xy-plane by the functions $x(u, v)$ and $y(u, v)$. The Jacobian is given by

$$\frac{\partial(x, y)}{\partial(u, v)} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}.$$

Curl and Divergence

Let $\mathbf{F}(x, y, z) = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$ be a vector field. The curl and divergence of $\mathbf{F}$ are given by:

$$\operatorname{curl} \mathbf{F} = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

$$\operatorname{div} \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$