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1. How many positive integers are there that are not larger than 1000 and are neither perfect squares nor perfect cubes?
 2. Let $F_k(n)$ be the number of partitions of the set $[n]$ into exactly k blocks in which each block contains two or more elements. Express the numbers $F_k(n)$ in terms of the Stirling numbers of the second kind.
 3. Let G be the union of k disjoint cycles of length r . How many automorphisms does G have?
 4. Let $K_{n,n}$ be the simple graph whose vertex set consists of two n -element vertex sets A and B . Two vertices are adjacent in this graph if and only if one vertex belongs to A and the other to B (so $K_{n,n}$ consists of all n^2 edges *between* A and B , but no edges within A or within B). How many *distinct* Hamiltonian cycles does $K_{n,n}$ contain?

Take care that you do not overcount. For example, let $K_{3,3}$ have vertex set $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. The cycles $1a2b3c1$, $a2b3c1a$, and $1c3b2a1$ all contain the same edges, so they are the same cycle. The second cycle is a rotation of the first, and the third cycle is a reflection of the first. The cycle $1b2a3c1$ is different than these, however, since it contains the edge (among others) $1b$ while the others do not.