

1. Select  $n + 1$  distinct numbers from the set  $\{1, 2, 3, \dots, 2n\}$ . Prove there will always be two among the selected integers whose greatest common divisor is 1.
2. Place 33 points inside a regular triangle of side length 1. Prove there will always be three points among them such that the triangle they define has area less than 0.03.
3. Let  $a_1 = 5$  and  $a_{n+1} = a_n^2$ . Prove that the last  $n$  digits of  $a_n$  are the same as the last  $n$  digits of  $a_{n+1}$ . (Hint: This is equivalent to proving  $a_{n+1} - a_n$  is divisible by  $10^n$ .)
4. Let  $n \geq 15$  be an integer. Prove that a square can be partitioned into  $n$  smaller squares (not necessarily all of the same size).