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M-Series for Sinx

$$\frac{f^n(0)\,x^n}{n!}$$

n	$f^n(x)$	$f^n(0)$
0	$\sin x$	0
1	$\cos x$	1
2	$-\sin x$	0
3	$-\cos x$	-1

$$\sin x = 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!}$$

$$= \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots$$

0 1 2 3

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\, x^{2n+1}}{(2n+1)!}$$

M Series $(1+x)^{-3}$

n	$f^n(x)$	$f^n(0)$
0	$(1+x)^{-3}$	1^{-3}
1	$-3(1+x)^{-4}$	$-3\cdot 1^{-4}$
2	$12(1+x)^{-5}$	$12 = 3\cdot 4$
3	$-60(1+x)^{-6}$	$-60 = 3\cdot 4\cdot 5$
4	$360(1+x)^{-7}$	$360 = 3\cdot 4\cdot 5\cdot 6$

$$(1+x)^{-3} = 1 - 3 + 12 - 60 + 360 - \dots$$

0 1 2 3 4

$$\sum_{n=0}^{\infty} (-1)^n\, n(n+2)$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n+2)!}{2!}$$

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Maclaurin series for sinx.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\sin x$	0
1	$\cos x$	1
2	$-\sin x$	0
3	$-\cos x$	-1

$$\sin x = \frac{0}{0!}x^0 + \frac{1}{1!}x^1 + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{-1}{7!}x^7$$
$$= 0 + x + 0 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$
$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

Maclaurin series for $\frac{1}{1-x}$

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$(1-x)^{-1}$	1
1	$-(1-x)^{-2}$	-1
2	$2(1-x)^{-3}$	2
3	$-6(1-x)^{-4}$	-6
4	$24(1-x)^{-5}$	24

$$\frac{1}{1-x} = \frac{1}{0!}x^0 + \frac{-1}{1!}x + \frac{2}{2!}x^2 + \frac{-6}{3!}x^3 + \frac{24}{4!}x^4$$
$$= 1 - x + x^2 - x^3 + x^4$$
$$= \sum_{n=0}^{\infty} (-1)^n x^n = \sum_{n=0}^{\infty} (-x)^n$$