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Binomial Series

Trevor + Kristin

(7) $\dfrac{x}{\sqrt{4+x^2}} = \dfrac{x}{\sqrt{4}\sqrt{1+x^2/4}}$

$\dfrac{x}{2}\left(1+x^2/4\right)^{-1/2}$

$\dfrac{x}{2}\sum_{n=0}^{\infty}\binom{-1/2}{n}\left(-\dfrac{x^2}{4}\right)^n$

$\dfrac{x}{2}\left(1+\dfrac{(-1/2)}{1!}(-x^2/4)+\dfrac{(-1/2)(-3/2)}{2!}(x^2/4)^2+\dfrac{(-1/2)(-3/2)(-5/2)}{3!}(x^2/4)^3\right)$

$\dfrac{x}{2}\left(1+\dfrac{x^2}{2^3}+\dfrac{1\cdot 3\, x^4}{2!\,2^6}-\dfrac{1\cdot 3\cdot 5\, x^6}{3!\,2^9}\right)$

$\left(\dfrac{x}{2}+\dfrac{x^3}{2^4}+\dfrac{1\cdot 3\, x^5}{2!\,2^7}-\dfrac{1\cdot 3\cdot 5\, x^7}{3!\,2^{10}}\right)$

Justin Ashler
Andy Chleborad
Tanner Best

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7. $$\frac{x}{\sqrt{4+x^2}} = \frac{x}{2\sqrt{1+\frac{x^2}{4}}}$$

$$= \frac{x}{2}\cdot\left(1+\left(\frac{x^2}{4}\right)\right)^{-1/2} \qquad x = x^2/4 \quad k=-1/2$$

$$= \frac{x}{2}\left(1+\frac{-1/2}{1!}\left(\frac{x^2}{4}\right)+\frac{(-1/2)(-3/2)}{2!}\left(\frac{x^2}{4}\right)^2+\frac{(-1/2)(-3/2)(-5/2)}{3!}\left(\frac{x^2}{4}\right)^3+\dots\right)$$

$$= \frac{x}{2}\left(1-\frac{x^2}{2}+\frac{1\cdot3x^4}{2!\,2^{10}}-\frac{1\cdot3\cdot5x^6}{3!\,2^{3\cdot5}}+\dots\right)$$

$$= \boxed{\frac{x}{2}\left(1+\sum_{n=1}^{\infty}(-1)^n\frac{1\cdot3\cdot5(2n-1)x^{2n+1}}{n!\,2^{3n+1}}\right)}$$

Adam Braegelman
Math 106 (Calc II)
4/16/14

Binomial Series

#2. $\frac{1}{\sqrt[3]{8-x}}$

$$= \frac{1}{(8-x)^{1/3}}$$

$$= \frac{1}{\left(8\left(1-\frac{x}{8}\right)\right)^{1/3}}$$

$$= \frac{1}{2}\left(1-\frac{x}{8}\right)^{-1/3}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\binom{-1/3}{n}\left(\frac{-x}{8}\right)^{n}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\binom{-1/3}{n}(-1)^{n}\left(\frac{x}{8}\right)^{n}$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\binom{-1/3}{n}(-1)^{n}\left(\frac{x^{n}}{2^{3n}}\right)$$

$$= \frac{1}{2}\left[1+\left(\frac{-1}{3}\right)\left(\frac{-x}{2^{3}}\right)+\left(\frac{-1}{3}\right)\left(\frac{-4}{3}\right)\left(\frac{1}{2!}\right)\left(\frac{x^{2}}{2^{6}}\right)+\left(\frac{-1}{3}\right)\left(\frac{-4}{3}\right)\left(\frac{-7}{3}\right)\left(\frac{1}{3!}\right)\left(\frac{-x^{3}}{2^{9}}\right)\right.$$

$$= \frac{1}{2}+\frac{1\cdot x}{3\cdot 2^{4}}+\frac{1\cdot 4x^{2}}{3^{2}\cdot 2!\,2^{7}}+\frac{1\cdot 4\cdot 7x^{3}}{3^{3}\,3!\,2^{10}}+\cdots$$

$$= \frac{1}{2}+\sum_{n=1}^{\infty}\frac{1\cdot 4\cdot 7\cdots(3n-2)}{3^{n}\cdot n!\cdot 2^{3n+1}}$$

Will R, Thomas S, Adam B

Binomial Series

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7. $\frac{x}{\sqrt{4+x^2}} = x\left(\frac{1}{(4+x^2)^{1/2}}\right) = \frac{1}{2}x\left(1+\frac{x^2}{4}\right)^{-1/2}$

$= \frac{x}{2}\left[1+\frac{-1/2}{1!}\left(\frac{x^2}{4}\right)^1 + \frac{(-1/2)(-3/2)}{2!}\left(\frac{x^2}{4}\right)^2 + \frac{(-1/2)(-3/2)(-5/2)}{3!}\left(\frac{x^2}{4}\right)^3 + \cdots\right]$

$= \frac{x}{2}\left[1 - \frac{x^2}{2^3 1!} + \frac{1\cdot 3 x^4}{2^6 2!} + \frac{1\cdot 3\cdot 5 x^6}{2^9 3!} - \cdots\right]$

$= \frac{x}{2} - \frac{x^3}{2^4 1!} + \frac{1\cdot 3 x^5}{2^7 2!} + \frac{1\cdot 3\cdot 5 x^7}{2^{10} 3!} - \cdots$

$= \frac{x}{2} + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1\cdot 3\cdot 5 \cdots (2n-1)}{n!\, 2^{3n+1}} x^{2n+1}$

Radius: $\left|\frac{-x^2}{4}\right| < 1$

$|x| < 2$