

Allison
Ben
Kristin
Andrew

pg. 703 pg. 3, 5, 13, 23

3. $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x \cdot n!}{(n+1)!} \right| = |x| \lim_{n \rightarrow \infty} \frac{n!}{(n+1)!}$$

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

divergent

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

divergent

$|x| < 1$

convergent when $|x| < 1$

divergent $x \in (-\infty, -1)$ and $(1, \infty)$

3. $\sum_{n=0}^{\infty} \frac{x^n}{5^{n+1}}$

Ratio Test

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{x^{n+1}}{5^{n+2}}}{\frac{x^n}{5^{n+1}}} \right| = \lim_{n \rightarrow \infty} \left| x \cdot \frac{5^{n+1}}{5^{n+2}} \right| = |x| \lim_{n \rightarrow \infty} \frac{5^{n+1}}{5^{n+2}} = |x| \cdot \frac{1}{5} = \frac{|x|}{5}$$

Converges when $-1 < x < 1$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/2}}$, convergent alt. series
 $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$, divergent p-series

Radius of convergence = 1

Interval of convergence = $[-1, 1)$

$$3) \sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{x^{n+1}}{\sqrt{n+1}}}{\frac{x^n}{\sqrt{n}}} \right| \rightarrow \lim_{n \rightarrow \infty} \frac{x^{n+1} \cdot \sqrt{n}}{x^n \cdot \sqrt{n+1}}$$

$$\lim_{n \rightarrow \infty} |x| \frac{\sqrt{n}}{\sqrt{n+1}}$$

$$|x| \cdot 1 < 1$$

radius = 1

$$[-1, 1)$$

$$\lim_{n \rightarrow \infty} \frac{(-1)^n}{\sqrt{n}} \quad \text{converges}$$

$$\lim_{n \rightarrow \infty} \frac{1^n}{\sqrt{n}} \quad \text{diverges}$$

$$5) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$$

$$\frac{(-1)^n x^{n+1}}{(n+1)^3} \cdot \frac{(-1)^{n-1} x^n}{n^3}$$

$$\frac{(-1)^n x^{n+1} n^3}{(n+1)^3 (-1)^{n-1} x^n}$$