3/11/14

Justin Ashley
Andy Chleborad
Tanner Bert

- Pg. 729 # 3, 5, 7, 11, 25, 27.

3. $$\sum_{n=1}^{\infty} \frac{1}{n^4} = \int_1^{\infty} \frac{1}{n^4}\,dn$$
$$= \left[\frac{-1}{3n^3}\right]_1^{\infty}$$
$$= \lim_{t\to\infty}\left(\frac{-1}{3t^3}\right) - \left(\frac{-1}{3(1)^3}\right)$$
$$= 0 + 1/3 = \boxed{1/3, \text{ Converges}}$$

5. $$\sum_{n=1}^{\infty} \frac{1}{3n+1} = \int_1^{\infty} \frac{1}{3n+1}\,dn \qquad u = 3n+1, \quad du = 3$$
$$= \frac{1}{3}\int_1^{\infty} \frac{1}{u}\,du$$
$$= \frac{1}{3}\Big[\ln|u|\Big]_1^{\infty}$$
$$= \frac{1}{3}\Big[\ln|3n+1|\Big]_1^{\infty}$$
$$= \frac{1}{3}\Big[\ln|\infty| - \ln|4|\Big] = \boxed{\infty, \text{ Diverges}}$$

25. $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p} = \int_2^{\infty} \frac{1}{n(\ln n)^p}\,dn \qquad u = \ln x, \quad du = 1/x\,dx$$
$$\int_2^{\infty} \frac{1}{u^p}\,du = \boxed{p > 1 \text{ to converge}}$$

27. $$\sum_{n=1}^{\infty} n(1+n^2)^p = \int_1^{\infty} n(1+n^2)^p\,dn \qquad u = 1+n^2, \quad du = 2n\,dn$$
$$= \int_1^{\infty} 1/2\, u^p \;\therefore\; \boxed{p < -1 \text{ to converge}}$$

Adam B., Will R., Thomas S.
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Integral Test

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#3. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

$$\int_1^{\infty} \frac{1}{x^4}\,dx$$
$$= \left[\frac{-1}{3} x^{-3}\right]_1^{\infty}$$
$$= \frac{-1}{3}\left(\frac{1}{x^3}\right)_1^{\infty}$$
$$= \frac{-1}{3} \lim_{t\to\infty} \frac{1}{t^3} - \frac{1}{1^3}$$
$$= \frac{-1}{3}(-1)$$
$$= 1/3 \text{, convergent}$$

#5. $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

$$\int_1^{\infty} \frac{1}{3x+1}\,dx$$
$$u = 3x+1$$
$$du = 3\,dx, \quad dx = \frac{1}{3}du$$
$$= \frac{1}{3}\int_1^{\infty} u^{-1}\,du$$
$$= \frac{1}{3}\left[\ln(u)\right]_1^{\infty}$$
$$= \frac{1}{3}\left[\ln(3x+1)\right]_1^{\infty}$$
$$= \frac{1}{3} \lim_{t\to\infty} \ln(3t+1) - \ln(3(1)+1)$$
$$= \infty \text{, divergent}$$

#7. $\sum_{n=1}^{\infty} \frac{n}{e^n}$

$$\int_1^{\infty} \frac{x}{e^x}\,dx$$

$u = x \qquad v = \frac{-1}{e^x}$

$du = 1\,dx \qquad dv = \frac{1}{e^x}$

$$= \left[\frac{-x}{e^x}\right]_1^{\infty} + \int_1^{\infty} e^{-x}\,dx$$

$$= \left[\frac{-x}{e^x} - e^{-x}\right]_1^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-t}{e^t} - \frac{1}{e^t}\right) - \lim_{t\to\infty}\left(\frac{-1}{e} - \frac{1}{e}\right)$$

$$= \lim_{t\to\infty}\frac{-t}{e^t} + \frac{2}{e}$$

$$\overset{L'R}{=} \lim_{t\to\infty}\frac{-1}{e^t} = 0$$

$$= \frac{2}{e}, \text{ convergent}$$

#25. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$

$$\int_2^{\infty} \frac{1}{x(\ln x)^p}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \int_2^{\infty} u^{-p}$$

If $p = 1$, $\int_2^{\infty} u^{-p} = \int_2^{\infty} u^{-1}$

$$= [\ln u] = \ln(\ln x)\Big]_2^{\infty}$$

$$= \lim_{t\to\infty} \ln(\ln t) - \ln(\ln 2) = \infty$$

If $p > 1$, $\int_2^{\infty} u^{-p}$

$$= \left[\frac{u^{-p+1}}{-p+1}\right]_2^{\infty} = \frac{1}{-p+1}\left[\ln(x)^{-p+1}\right]_2^{\infty}$$

$$= \frac{1}{-p+1}\lim_{t\to\infty}\left(\ln(t)^{-p+1} - \ln(2)^{-p+1}\right)$$

converges

Andrew
Allison
Kristin
Ben

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3.) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

$\int_1^\infty \frac{1}{n^4}$

$= \int_1^\infty n^{-4}$

$= -\frac{1}{3} n^{-3} \Big|_1^\infty$

$\lim_{t\to\infty} \frac{-1}{3t^3} - \frac{-1}{3(1)^3}$

$\lim_{t\to\infty} \frac{-1}{3t^3} + \frac{1}{3}$

$= 0 + \frac{1}{3} = \frac{1}{3}$ convergent

5.) $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

$\int_1^\infty \frac{1}{3n+1}$

$= \frac{1}{3} \ln|3x+1| \Big|_1^\infty$

$= \lim_{t\to\infty} \frac{1}{3} \ln|3t+1| - \frac{1}{3}\ln|4|$

$= \infty$ diverges

7.) $\sum_{n=1}^{\infty} \cancel{x}e^{-\cancel{x}}$

let $u = \cancel{x}$ $\quad v = -e^{-n}$

$du = dx$ $\quad dv = e^{-n}$

$\int_1^\infty (x)(-e^{-n}) - \int(-e^{-n})(dx)$

$= \int_1^\infty -xe^{-\cancel{x}} + \int e^{-n} dx$

$= \int_1^\infty -xe^{-\cancel{x}} - e^{-x}$

$= -xe^{-x} - e^{-x} \Big|_1^\infty$

$\lim_{t\to\infty} \underbrace{(-te^{-x} - e^{-t})}_{0} - (-1e^{-1} - e^{-1})$

$= \frac{+2}{e}$

25.) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$

$\int_2^{\infty} \frac{1}{x(\ln x)^p}\, dx$

$u = \ln x$

$du = \frac{1}{x} dx \qquad x\,du = dx$

$= \int_1^{\infty} \frac{1}{u^p}$

$= \int_1^{\infty} \frac{1}{x^p}\, dx = \int_1^{\infty} x^{-p}\, dx$

$= \left[\frac{x^{-p+1}}{-p+1} \right]_1^{\infty}$

$= \frac{1}{-p+1} \left[x^{-p+1} \right]_1^{\infty}$

$= \frac{1}{-p+1} \lim_{t \to \infty} \frac{1}{x^{p-1}} - \frac{1}{1^p}$

If $p > 1$, then $p - 1 > 0$

so $\lim_{t \to \infty} \frac{1}{t^{p-1}} = 0$ convergent

If $p < 1$, then $p - 1 < 0$

so $\lim_{t \to \infty} \frac{1}{t^{p-1}} = \lim_{t \to \infty} t^{1-p} = \infty$ ~~convergent~~ divergent