Will R, Adam B, Thomas S.

Trigonometric Integrals

Pg 488, #1, 7, 9, 21, 23, 27

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1. Did in class

7. $\int_0^{\pi/2} \cos^2 x \, dx$

$= \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2x) dx$

$= \frac{1}{2} \int_0^{\pi/2} (1 + \cos 2x) dx$

$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)\Big|_0^{\pi/2}$

$= \frac{1}{2}\left(\frac{\pi}{2}\right) - 0$

$= \boxed{\frac{\pi}{4}}$

9. $\int_0^{\pi} \sin^4(3t) \, dt$

$= \int_0^{\pi} \sin^2(3t)\sin^2(3t) dt$

$= \int_0^{\pi} \frac{1}{2}(1-\cos 6t) \cdot \frac{1}{2}(1-\cos 6t) dt$

$= \frac{1}{4}\int_0^{\pi} (1-\cos 6t)^2 dt$

$= \frac{1}{4}\int_0^{\pi} (1 - 2\cos 6t + \cos^2 6t) dt$

$= \frac{1}{4}\int_0^{\pi} (1 - 2\cos 6t + \frac{1}{2}(1 + \cos 12t)) dt$

$= \frac{1}{4}\int_0^{\pi} (\frac{3}{2} - 2\cos 6t + \frac{\cos 12t}{2}) dt$

$= \frac{1}{4}\left(\frac{3}{2}t - \frac{1}{3}\sin 6t + \frac{1}{24}\sin 12t\right)\Big|_0^{\pi}$

$= \boxed{\frac{3\pi}{8}}$

21. $\int \sec^2 x \tan x \, dx$ $\quad u = \tan x \quad du = \sec^2 x \, dx$

$= \int \sec x \sec x \tan x \, dx$ $\quad u = \sec x \quad du = \sec x \tan x \, dx$

$= \int u \, du$

$= \frac{1}{2}u^2 + C$

$= \boxed{\frac{1}{2}\sec^2 x + C}$

$\int u \, du$

$= \frac{1}{2}u^2 + C$

$= \boxed{\frac{1}{2}\tan^2 x + C}$

23. $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1) dx$

$= \int \sec^2 x \, dx - \int 1 dx$

$= \boxed{\tan x - x + C}$

27. $\int_0^{\pi/3} \tan^5 x \sec^4 x \, dx$ $\quad u = \tan x$

$= \int_0^{\pi/3} u^5 \sec^2 x \, du$ $\quad du = \sec^2 x \, dx$

$= \int_0^{\pi/3} u^5 (1 + \tan^2 x) du$

$= \int_0^{\pi/3} u^5 (1 + u^2) du$

$= \int_0^{\pi/3} (u^5 + u^7) du$

$= \frac{u^6}{6} + \frac{u^8}{8}$

$= \frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} \Big|_0^{\pi/3}$

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Jan 23
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9.) $\int_0^{\pi} \sin^4(3t)\,dt$

$\int_0^{\pi} \sin^2(3t)\,\sin^2(3t)\,dt$

$\int_0^{\pi} \frac{1}{2}(1-\cos 6t)\,\frac{1}{2}(1-\cos 6t)$

$\frac{1}{4}\int_0^{\pi} (1-\cos 6t)^2\,dt$

$\frac{1}{4}\int_0^{\pi} 1-2\cos(6t)+\cos^2(6t)\,dt$

$\frac{1}{4}\int_0^{\pi} 1-2\cos(6t)+\frac{1}{2}(1+\cos(12t))\,dt$

$\frac{1}{4}\int_0^{\pi} \frac{3}{2}-2\cos(6t)+\frac{1}{2}\cos(12t)\,dt$

$\frac{1}{4}\left(\frac{3t}{2}+\frac{1}{3}\sin(6t)-\frac{1}{24}\sin(12t)\right)\Big|_0^{\pi}$

$\frac{3\pi}{8}$ $\quad \frac{1}{4}\left(\left(\frac{3\pi}{2}+0-0\right)-(0)\right)$

$\frac{3\pi}{8}$

21.) $\int \sec^2 x \tan x \, dx$

$u - \tan$

$du - \sec^2 x \, dx$

$\int u \, du$

$\frac{u^2}{2}$

$\frac{\tan^2 x}{2} + C$

23.) $\int \tan^2 x \, dx$

$\int \sec^2 x - 1 \, dx$

$\tan x - x + C$

27.) $\int_0^{\frac{\pi}{3}} \tan^5 x \sec^4 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int_0^{\frac{\pi}{3}} u^5 (u^2+1) \, du$

$\int_0^{\frac{\pi}{3}} u^7 + u^5 \, du$

$\frac{u^8}{8} + \frac{u^6}{6} \Big|_0^{\frac{\pi}{3}}$

$\frac{\tan^8 x}{8} + \frac{\tan^6}{6} \Big|_0^{\frac{\pi}{3}}$

14.625

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1.) $\int \sin^3 x \cos^2 x \, dx$

$$= \int \sin^2 x \cos^2 x (\sin x)\, dx$$
$$= \int \sin x \, (1-\cos^2 x) \cos^2 x \, dx$$
$$= \int \sin x \, (1-v^2)\, v^2 \, du$$
$$= -\int (v^2 - v^4)\, du \qquad v = \cos x$$
$$= \frac{-v^3}{3} - \frac{v^5}{5} + C \qquad dv = -\sin x \, dx$$
$$= \frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

7.) $\int_0^{\pi/2} \cos^2\theta \, d\theta$

$$= \int_0^{\pi/2} \frac{1}{2}(1+\cos 2x)\, dx$$
$$= \frac{1}{2}\int_0^{\pi/2} \left(x + \frac{\sin 2x}{2}\right) dx$$
$$= \frac{x}{2} + \frac{\sin 2x}{4} \Big|_0^{\pi/2}$$
$$= \pi/4$$

9.) $\int_0^{\pi} \sin^4(3t)\, dt$

$$\int \cos^2 x = \frac{1}{2}(1+\cos 2(6t))$$
$$\int \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(12t)$$
$$= \frac{1}{2}t + \frac{1}{12}\sin(12t)$$

$$= \int_0^{\pi} \left(\frac{1}{2}(1-\cos(6t))\right)\left(\frac{1}{2}(1-\cos(6t))\right) dt$$
$$= \frac{1}{4}\int_0^{\pi} (1-\cos(6t))(1-\cos(6t))\, dt$$
$$= \frac{1}{4}\int_0^{\pi} (1 - 2\cos 6t + \cos^2 6t)\, dt$$
$$= \frac{1}{4}\left(t - \frac{1}{3}\sin 6t + \frac{1}{2}t + \frac{1}{24}\sin(12t)\right)\Big|_0^{\pi}$$
$$= \frac{1}{4}\left[\left(\pi - \frac{1}{3}\sin(6\pi) + \frac{1}{2}(\pi) + \frac{1}{24}\sin(12(\pi))\right) - \left(0 - \frac{1}{3}\sin 6(0) + \frac{1}{2}(0) + \frac{1}{24}\sin(12(0))\right)\right.$$
$$= \frac{1}{4}\left[\pi - 0 + \frac{1}{2}\pi + 0\right] - 0$$
$$= \frac{\pi}{4} + \frac{\pi}{2}$$
$$= \boxed{3\pi/8}$$

21.) $\int \sec^2 x \tan x \, dx$

$u = \tan x$

$= \int \frac{du}{\sec^2 x} = \frac{\cancel{\sec^2 x}\, dx}{\cancel{\sec^2 x}}$

$= \int \frac{du}{\sec^2 x} = dx$

$= \int \cancel{\sec^2 u}\, u \left(\frac{du}{\cancel{\sec^2 u}}\right)$

$= \int u \, du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\tan^2 x + C$

23.) $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1)\, dx$

$= \tan x - x + C$

27.) $\int_0^{\pi/3} \tan^5 x \sec^4 x \, dx$