Nebraska Wesleyan University — Formulas
Math 106 — Spring 2014
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Trigonometric Identities

$$\sin(2x) = 2\sin x \cos x \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\cos(2x) = \cos^2 x - \sin^2 x \qquad \sin^2 x = \frac{1-\cos(2x)}{2}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

Error Bounds for Approximate Integration

Let E_M and E_T denote the error in estimating an integral with the Midpoint and Trapezoid Rules, respectively. The use of these rules to evaluate $\int_a^b f(x)\,dx$ on n subintervals yields

- $|E_M| \leq \frac{K(b-a)^3}{24n^2}$ and
- $|E_T| \leq \frac{K(b-a)^3}{12n^2}$,

where K is a constant satisfying $|f''(x)| \leq K$.

Hydrostatic Force

The hydrostatic force of water against a surface with an area of A m^2 at a depth of d m is approximately $9800dA$ Newtons.

Polar Areas

The area bounded by the polar curve $r(\theta)$ traced out from $\theta = a$ to $\theta = b$ is given by

$$\frac{1}{2}\int_a^b r(\theta)^2 d\theta.$$

Taylor Series

If f has a power series representation at $x = a$, then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}.$$

Taylor's Theorem

If $f(x)$ is $(n+1)$-times differentiable, then $f(x) = T_n(x) + R_n(x)$, where

$T_n(x)$ is the n^{th} degree Taylor polynomial centered at a, and

$R_n(x) = \dfrac{f^{(n+1)}(k)(x-a)^{n+1}}{(n+1)!}$ for some k between a and x.

A useful bound is

$$|R_n(x)| \leq \frac{|x-a|^{n+1}}{(n+1)!} \max_{a \leq k \leq x} |f^{(n+1)}(k)|.$$

Common Maclaurin Series

Function	Maclaurin Series	Convergence
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$-\infty < x < \infty$
$sinx$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$-\infty < x < \infty$
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$-\infty < x < \infty$
$(1+x)^k$	$1 + \sum_{n=1}^{\infty} k(k-1)\cdots(k-n+1)\frac{x^n}{n!}$ (k any real number)	$-1 < x < 1$