Nebraska Wesleyan University	Test 3B
Math 060	Spring 2014
Instructor: Austin Mohr	Solutions

Please write only your name on this sheet.
Write all work and answers on the blank sheets.
Only attempt problems for which you have not already received a grade of "Master".
Use algebra or calculus techniques to solve these problems. Appealing to a graph will not earn any credit.

9. The value of a certain savings account (in dollars) can be modeled by $V(t) = 1000e^{0.07t}$, where t is the number of years since the creation of the account.
   (a) What is the value of the account after five years?
   (b) How long does it take for this account to double its value?
   (c) At what rate (in dollars per year) is the account growing at the end of the third year?

   Solution: For part (a), we know $t = 5$, so the value of the account is $P(5) = 1000e^{0.07 \cdot 5} \approx \$1419.07$.

   For part (b), we know the value of the account is \$2,000, so we solve

$$\begin{aligned} 2000 &= 1000e^{0.07t} \\ 2 &= e^{0.07t} \\ \ln 2 &= 0.07t \\ \frac{\ln 2}{0.07} &= t \\ 9.9 &\approx t. \end{aligned}$$

   The value of the account will double in value after approximately 9.9 years.

   For part (c), we first compute the derivative $P'(t) = 70e^{0.07t}$. To determine the rate of change at the third year, we compute $P'(3) = 70e^{0.07 \cdot 3} \approx 86.36$ dollars per year.

10. In each of the following situations, decide whether an exponential function is an appropriate model. If it is, provide the exponential function $f(t)$ and clearly state the units of t and $f(t)$. If an exponential function is not appropriate, explain why not.
    (a) Approximately 4.7 billion bottles of water were sold in the year 2000. Sales continued to grow at a rate of 9.3% per year.
    (b) A company employs thirty workers and plans to hire two new employees each month.
    (c) You have a 100 mg sample of Strontium-90, which has a half-life of roughly twenty-five years.

Solution: Part (a) calls for an exponential function since the sales grow at a constant percentage rate. We'll let the units of $f(t)$ be in billions of bottles and t be years, so that an partial solution is $f(t) = 4.7e^{kt}$. We can determine k by finding another point of data. Since sales grow by 9.3% each year, another point of data is $(1, 5.16)$. We can now solve

$$\begin{aligned} 5.16 &= 4.7e^{k \cdot 1} \\ \frac{5.16}{4.7} &= e^k \\ \ln\left(\frac{5.16}{4.7}\right) &= k \\ 0.093 &\approx k. \end{aligned}$$

A good model is therefore $f(t) = 4.7e^{0.093t}$.

Part (b) has a constant rate of change, so it is better suited for a linear function.

Part (c) gives a half-life, which implies an exponential model is appropriate. As in part (a), we start with a partial model $f(t) = 100e^{kt}$, where $f(t)$ gives the remaining mass (in grams) t years from now. Another data point we can use is $(25, 50)$, since the half-life is twenty-five years. We can now solve

$$\begin{aligned} 50 &= 100e^{k \cdot 25} \\ \frac{1}{2} &= e^{25k} \\ \ln\left(\frac{1}{2}\right) &= 25k \\ \frac{\ln\left(\frac{1}{2}\right)}{25} &= k \\ -0.028 &\approx k. \end{aligned}$$

A good model is therefore $f(t) = 100e^{-0.028t}$.

11. A particle is currently traveling at a rate of 5 $\frac{\text{m}}{\text{sec}}$. We will denote its current position as 0 m. From this point, it begins to accelerate according to the function $a(t) = 4t$, where t is measured in seconds. Construct a function $s(t)$ that gives the particles position t seconds after this moment.

    Solution: We use indefinite integration to write

$$\begin{aligned} v(t) &= \int s(t)dt \\ &= \int 4tdt \\ &= 2t^2 + C. \end{aligned}$$

The initial velocity condition tells that $v(0) = 5$, so we can solve

$$\begin{aligned} 5 &= 2 \cdot 0^2 + C \\ 5 &= C. \end{aligned}$$

Therefore, $v(t) = 2t^2 + 5$.

Using indefinite integration again, we now write

$$\begin{aligned} s(t) &= \int v(t)dt \\ &= \int (2t^2 + 5)dt \\ &= \frac{2}{3}t^3 + 5t + C. \end{aligned}$$

The initial position condition tells that $s(0) = 0$, so we can solve

$$\begin{aligned} 0 &= \frac{2}{3}0^3 + 5 \cdot 0 + C \\ 0 &= C. \end{aligned}$$

Finally, we have $s(t) = \frac{2}{3}t^3 + 5t$.

12. A company estimates that its profit will grow continuously at a rate (in thousands of dollars per day) given by the function $P'(t) = 2e^{0.01t}$, where t is the number of days the business has been open. What is the total profit the company predicts for the first thirty days of business?

    Solution: The total profit is given by $\int_0^{30} 2e^{0.01t}dt$. To evaluate this, we first find an antiderivative $F(t) = 200e^{0.01t}$. Now,

$$\begin{aligned} \int_0^{30} 2e^{0.01t}dt &= F(30) - F(0) \\ &= 200e^{0.01 \cdot 30} - 200e^{0.01 \cdot 0} \\ &\approx 69.972. \end{aligned}$$

    The total profit for the first thirty days is approximately \$69,972.