Nebraska Wesleyan University
Math 060
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Test 1
Spring 2014
Solutions

1. In each of the following situations, decide whether a linear function is an appropriate model. If it is, provide the linear function $f(t)$ and clearly state the units of t and $f(t)$. If a linear function is not appropriate, explain why not.

   (a) A tank contains 40 gallons of water. The water begins to flow out at a constant rate of 5 gallons per minute.

   Solution: Since the water flow is constant, a linear model $f(t) = 40 - 5t$ is appropriate. In this model, $f(t)$ measures the current volume (in gallons) of the tank t minutes after the flow begins.

   (b) The initial mass of a radioactive element is 50 grams and decays by half each second.

   Solution: The rate of change is not constant, so a linear model is not appropriate. For instance, the mass decreases by 25 grams in the first second but only 12.5 grams in the next second.

   (c) The depth of the snow at noon one day is 3 inches. Snow falls at a constant rate for eight hours, at which time the depth of the snow is 7 inches.

   Solution: The snow fall is constant, but we need to determine that rate in order to construct the model. We can use the two data points to compute $\frac{7-3}{8-0} = \frac{1}{2}$. Putting this together with the initial value, we can use the model $f(t) = 3 + \frac{1}{2}t$, where $f(t)$ measures the current depth of snow (in inches) t hours past noon.

2. Compute each limit (or conclude that it does not exist) using the specified method.

   (a) $\lim_{x \to 0} 3x + 2$, numerical

   Solution: We need to construct two tables: one with x approaching 0 from the left and one from the right.

x	f(x)
0.1	2.3
0.01	2.03
0.001	2.003

x	f(x)
-0.1	1.7
-0.01	1.97
-0.001	1.997

   In either case, it is evident that $f(x)$ is approaching 2. Therefore, $\lim_{x\to 0} 3x + 2 = 2$.

(b) $\lim_{x\to\infty} \frac{1}{x}$, graphical

Solution: From the graph of $y = \frac{1}{x}$, we see that the curve approaches $y = 0$ as x gets larger. Therefore, $\lim_{x\to\infty} \frac{1}{x} = 0$.

(c) $\lim_{x\to 0} \frac{2x^2+1}{x^3-1}$, continuity

Solution: Rational functions are continuous on their domains. Since evaluation at $x = 1$ does not result in division by 0, we may conclude $\lim_{x\to 0} \frac{2x^2+1}{x^3-1} = \frac{2\cdot 0^2+1}{0^3-1} = -1$.

3. Compute the derivative $f'(x)$ of each of the following functions using the specified method.

   (a) $f(x) = 2x + 1$, difference quotient

   Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(2(x+h)+1) - (2x+1)}{h} \\ &= \lim_{h\to 0} \frac{2x + 2h + 1 - 2x - 1}{h} \\ &= \lim_{h\to 0} \frac{2h}{h} \\ &= \lim_{h\to 0} 2 \\ &= 2. \end{aligned}$$

   (b) $f(x) = \sqrt{x} + \frac{2}{x}$, power/sum/difference rules

   Solution: Write $f(x) = x^{\frac{1}{2}} + 2x^{-1}$, so that

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2} \\ &= \frac{1}{2\sqrt{x}} - \frac{2}{x^2}. \end{aligned}$$

4. A particle accelerates through space in a straight line. The function $s(t) = 3t^2$ gives the distance (in meters) of the particle from its starting point after traveling for t seconds.

   (a) What is the particle's acceleration after 4 seconds?

   (b) What is the particle's velocity after 4 seconds?

   (c) What is the particle's distance after 4 seconds?

Solution: We will answer these questions all at once. Velocity is the rate of change of position, so $v(t) = s'(t) = 6t$. Acceleration is the rate of change of velocity, so $a(t) = v'(t) = 6$. The particle's distance at $t = 4$ is therefore $s(4) = 48$ meters. The particle's velocity at $t = 4$ is $v(4) = 24$ meters per second. The particle's acceleration at $t = 4$ is $a(4) = 6$ meters per second squared.